

ON THE SCHEME DEPENDENCE OF THE ELECTROWEAK RADIATIVE CORRECTIONS

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Abstract

We study the scheme dependence of the two-loop expression for the ρ -parameter of the Standard Model in the heavy t -quark mass limit $m_t \gg m_w$. In the \overline{MS} -scheme the two-loop electroweak correction to the $\mathcal{O}(m_t^2)$ term in $\Delta\rho$ is found greater than the QCD α_s -correction.

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To confront current and forthcoming precision electroweak (EW) data, one-loop, and recently, two-loop contributions to various EW parameters have been calculated. Accurate information about the EW radiative corrections is required to determine the unknown parameters of the EW Lagrangian or obtain constraints on possible extensions of the Standard Model (SM). When evaluating higher-order radiative corrections it is important to know the optimal parameter of the perturbative expansion, so that the coefficients in power series of that parameter are small. To achieve this one naturally looks for the appropriate renormalization scheme. The \overline{MS} -scheme may be advantageous for the higher-order corrections to the off-shell quantities, since one can solve renormalization group equations with non-zero masses in that scheme [1]. In this note we consider the ρ -parameter of the SM. Since the t -quark is reportedly [2] twice as heavy as the W -boson the perturbative series for ρ is predominantly an expansion in powers of α_t ($\alpha_t \equiv g_t^2/4\pi$ with g_t being the top Yukawa coupling constant) and QCD coupling constant α_s . For the currently discussed value $m_t \simeq 174 GeV$ the two coupling constants are rather close: $\alpha_t \simeq .08$ and $\alpha_s(m_t) \simeq 0.11$. Below we convert the known two-loop expression for ρ into the \overline{MS} -scheme and find that in terms of the running coupling constants $\alpha_t(m_t)$ and $\alpha_s(m_t)$ the EW $\mathcal{O}(\alpha_t^2)$ corrections are greater than the QCD $\mathcal{O}(\alpha_t\alpha_s)$ ones. We also obtain a relation between the physical and \overline{MS} - scalar selfcoupling constant λ and present the \overline{MS} -expression for the recently calculated $\mathcal{O}(\lambda^2)$ -correction to the decay of Higgs boson into fermion-antifermion pair.

The ρ -parameter is the ratio of the amplitudes of neutral and charged weak currents at low energies. In the SM ρ differs from its tree level value $\rho = 1$ due to non-zero mass splitting within the fermionic doublets [3]. The largest correction comes from the t -quark. For $m_t \gg m_w$, ρ is given by the ratio of the W - and Z -boson propagators at zero momentum:

$$\rho = \frac{1 + \Pi_w(k^2 = 0)/m_w^2}{1 + \Pi_z(k^2 = 0)/m_z^2} \quad (1)$$

where $\Pi_{w,z}(k^2)$ are the transverse parts of the polarization operators of W - and Z -bosons.

For $m_b \ll m_t$ one obtains for $\Delta\rho \equiv 1 - 1/\rho$:

$$\Delta\rho = N_c \frac{\alpha_t^*}{8\pi} \left[1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 + 3) + \frac{\alpha_t^*}{8\pi} \rho^{(2)} \right] \quad (2)$$

where $N_c = 3$ is the number of colors. The QCD-correction $\mathcal{O}(\alpha_t\alpha_s)$ comes from [4]. Possible form of the further QCD-corrections were discussed recently in [5]. The EW correction $\mathcal{O}(\alpha_t^2)$

is determined by $\rho^{(2)}$, which is a function of the ratio $r = m_H/m_t$ of the Higgs boson mass to the t -quark mass. It was computed numerically in [7] and analytically in [8]. The expression (2) is obtained in the on-shell momentum subtraction scheme (marked by the superscript "s")¹. The parameter α_t^* is expressed in terms of the measured quantities

$$\alpha_t^* = \frac{G_\mu^*}{\sqrt{2}} \frac{(m_t^*)^2}{\pi}, \quad (3)$$

where the Fermi-decay coupling constant $G_\mu^* = 1.16639(2) \cdot 10^{-5} \text{GeV}^{-2}$ [6] is defined in such a way that pure electromagnetic corrections do not contribute to it. As far as the virtual heavy top and Higgs boson contributions are concerned, G_μ is determined by the W -boson polarization operator:

$$G_\mu = \frac{1}{\sqrt{2} v^2 (1 + \Pi_w(k^2 = 0)/m_w^2)} \quad (4)$$

v being the v.e.v. of the Higgs doublet: $v = 2m_w/g_w$.

$\Delta\rho$ is a physical quantity. It is made finite by the renormalization of the Lagrangian parameters - masses and coupling constants. The quark mass in (3) depends on the scheme in which both EW and QCD calculations were performed, while the G_μ according to (4) depends on the scheme of the EW calculations. Specifying the scheme one chooses a relation between the bare Lagrangian parameters (marked by the superscript "B") and the renormalised ones. For instance, the bare quark propagator $S(k)$ parametrized near the pole as

$$S^{-1}(\hat{k} \rightarrow m_t^*) = X_L^* \hat{k} P_L + X_R^* \hat{k} P_R - m_t^B X_m^* \quad (5)$$

with $P_{L,R} = (1 \pm \gamma_5)/2$, determines the physical mass in terms of the bare one: $m_t^* = m_t^B X_m^* / \sqrt{X_L^* X_R^*}$. Changing the on-shell subtracted X^* 's for the X^{MS} 's one obtains a relation between the pole-quark mass m_t^* and \overline{MS} -mass m_t . The QCD-part of the function $m_t^*[m_t]$ was computed on the one-loop level in [9] and on the two-loop level in [10]. The one-loop relation

$$m_t^* = m_t \left(1 + \frac{4}{3} \frac{\alpha_s}{\pi} + \frac{\alpha_s}{\pi} \ln(\mu^2/m_t^2) \right) \quad (6)$$

with the usual choice of the normalization point $\mu = m_t$ leads to the following form of the $\mathcal{O}(\alpha_t \alpha_s)$ - correction:

$$\Delta\rho = N_c \frac{\alpha_t^*}{8\pi} \left[1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 - 9) + \frac{\alpha_t^*}{8\pi} \rho^{(2)} \right] \quad (7)$$

¹We will not be sensitive to the scheme dependence of α_s

The α_t^* in (7) reminds us that the EW $\mathcal{O}(\alpha_t^2)$ -correction is still given in terms the on-shell quantities according to (3). The rather small coefficient of the $\mathcal{O}(\alpha_s)$ -term in (7) and possibly [2] close numerical values of α_s and α_t indicate that the $\mathcal{O}(\alpha_s\alpha_t)$ - and $\mathcal{O}(\alpha_t^2)$ -corrections are of the same order of magnitude. To clarify this point we now obtain a complete \overline{MS} -expression for $\Delta\rho$ in the SM.

The two-loop EW calculations of $\Delta\rho$ are done in terms of the bare coupling constant α_t^B and then, in the course of renormalization, are usually expressed in terms of the on-shell renormalized Yukawa coupling constant α_t^* :

$$\alpha_t^* \equiv \alpha_t^B (X_m^*)^2 / X_w^* X_L^* X_R^* \quad (8)$$

To obtain (8) one uses (4) with $X_w^* \equiv 1 + \Pi_w^B(k^2 = 0)/m_w^2$ and $m_t^B \equiv g_t^B v^B / \sqrt{2}$. X_w is the renormalization constant of the W -boson mass: $m_w^{*,MS} = m_w^B \sqrt{X_w^{*,MS}}$ in the limit $m_t \gg m_w$ [11]. Due to the following Ward identity [7] it is related to the self-energy of the corresponding charged Goldstone bosons $\Pi_\phi(k^2)$:

$$\Pi_w(k^2 = 0)/m_w^2 = \lim_{k^2 \rightarrow 0} \Pi_\phi(k^2)/k^2 \quad (9)$$

In the \overline{MS} -scheme meanwhile one has:

$$\alpha_t^{MS} \equiv \alpha_t^B / X_w^{MS} X_L^{MS} X_R^{MS} \quad (10)$$

Evaluating the one-loop diagrams of self-energies $S^{-1}(k)$ of the t -quark and the charged Goldstone boson $\Pi_\phi(k^2)$ we obtain the factors $X^{*,MS}$, which determine the corresponding schemes. For the ratio of the physical α_t^* and \overline{MS} -parameter α_t^{MS} one has:

$$\frac{\alpha_t^*}{\alpha_t^{MS}} = \frac{X_w^{MS} X_L^{MS} X_R^{MS}}{X_w^* X_L^* X_R^*} (X_m^*)^2 \quad (11)$$

The \overline{MS} -bare renormalization constants have generic form $X^{MS} = 1 + \mathcal{O}(1/\epsilon - \gamma_E + \ln(4\pi))$, where ϵ and γ_E are the conventional parameters of the dimensional regularization. The factors X^* have nontrivial finite parts and a dependence on the normalization point μ^2 . Note that $X_m^{MS} = 1$, since there is no ultraviolet-infinite renormalization of the mass counterterm at least on the one-loop level.

The calculation of the $\mathcal{O}(\alpha_t)$ -corrections is done with the conventional EW Lagrangian in a renormalizable gauge in the gaugeless (gauge-invariant) limit $\alpha_w = 0$ [7], [8]. The

corresponding theory is a linear σ -model of a scalar doublet Φ interacting with a doublet of fermions (t, b) , and only the t -quark is relevant for $m_t \gg m_b$. For the EW relation between the pole mass m_t^* and the \overline{MS} -mass m_t we find:

$$m_t^* = m_t \left(1 + \frac{\alpha_t^{MS}}{8\pi} \left[\Delta m(r) - \frac{3}{2} \ln(\mu^2/m_t^2) \right] \right) \quad (12)$$

with

$$\Delta m(r) = \int_0^1 dx (2-x) \ln(r^2(1-x) + x^2) - \frac{1}{2} \quad (13)$$

In contrast to QCD, the very notion of the \overline{MS} -mass m_t in the EW theory is not determined entirely by the prescriptions of the minimal subtraction scheme. It depends on the value of v chosen as a parameter of the calculations: $m_t = g_t v$. Eqn. (12) corresponds to the v , incorporating the tadpole contributions (Fig. 1), which are nonzero in the \overline{MS} -scheme (the details are in [12]):

$$v^2 = v_{MS}^2 \left(1 + 6\hat{\lambda} - \frac{2N_c}{\hat{\lambda}} \left(\frac{\alpha_t}{8\pi} \right)^2 \right) \quad (14)$$

where $\hat{\lambda} = \lambda/(16\pi^2)$, λ being a scalar self-coupling constant and v_{MS} - parameter of the \overline{MS} -calculations. For the relation (11) between the on-shell- and the \overline{MS} -Yukawa coupling constants one obtains (see Fig. 2):

$$\alpha_t^* = \alpha_t^{MS} \left[1 + \frac{\alpha_t^{MS}}{8\pi} \left\{ 2\Delta m(r) - N_c - \frac{1}{2}r^2 - (3 + 2N_c) \ln(\mu^2/m_t^2) \right\} \right] \quad (15)$$

The relation (15) between dimensionless quantities does not require a calculation of the tadpole diagrams, corresponding to the shift (14) in v . The term $\sim r^2 \alpha_t^{MS}$ in (15) is essentially the scalar self-coupling constant λ originating from the renormalization of G_μ according to eq.(4). Thus we find a $\mathcal{O}(G_\mu^2 m_t^2 m_H^2)$ -correction in the total \overline{MS} -expression for $\Delta\rho$, which reads as

$$\Delta\rho^{MS} = N_c \frac{\alpha_t}{8\pi} \left[1 - \frac{2}{9} \frac{\alpha_s}{\pi} (\pi^2 - 9) + \frac{\alpha_t}{8\pi} \left(\rho^{(2)} + 2\Delta m(r) - N_c - \frac{1}{2}r^2 \right) \right] \quad (16)$$

where both α_t and α_s are \overline{MS} -parameters normalized at $\mu^2 = m_t^2$. Because of the smallness of the coefficient in the $\mathcal{O}(\alpha_s \alpha_t)$ -term the correction is dominated by the $\mathcal{O}(G_\mu^2 m_t^2 m_H^2)$ -term ($r^2/2$ -term in (15)) which is not present in the on-shell renormalized $\Delta\rho^{MOS}$.

To fix the electroweak contribution, we take for illustration, $m_t = 174 GeV$ (then $\alpha_t^* = .076$ and $\alpha_s(m_t) = .11$) and shift $\alpha_t^* \rightarrow \alpha_t$ for $m_H = 300 GeV$ Higgs boson. The results

are presented in two graphs, Fig. 3 and Fig. 4. From these figures, one can see that both the EW term and the QCD term have the same sign (negative) and the EW contribution is substantially larger than the QCD contribution, by a factor of 6 for light Higgs to a factor of 15 for a Higgs mass of slightly more than 1 TeV ($r = 6$). For 1 TeV Higgs, the two-loop EW correction is dominated by the $\mathcal{O}(G_\mu m_t^2 m_H^2)$ -term ($\frac{1}{2}r^2$ -term in eq.(16)). In the approximation we considered ($m_t, m_H \gg m_w$), $\Delta\rho$ as defined in (1) is a physical quantity. Thus, if calculated to all orders, its value should be independent of the reparametrization involved in the change of the schemes. In finite order there are unequal truncation errors, but these should be small if perturbation theory is good in both schemes. We fix α_t by inverting (15) at some conventionally chosen value of r . Since many SM loop correction calculations are done with an illustrative value $m_H = 300\text{ GeV}$, we make the conversion at $r = 300/174$. Then we obtain a nontrivial r -dependent scheme dependence (Fig. 4). Again one can see the domination for large r of the $r^2/2$ -term in (16), introduced by the transformation (15).

Consider now radiative corrections to the heavy Higgs boson decay into fermion-antifermion pair [13]. The two-loop corrections $\mathcal{O}(\lambda^2)$ have been computed recently with two different answers [14] and [15]. The later calculation [15] gives the following correction $\Delta\Gamma$ to the decay rate $\Gamma(H \rightarrow f\bar{f})$, performed within the momentum-subtracted on-shell scheme:

$$\Delta\Gamma = 1 + 2.1172\hat{\lambda}^* - 19.4483(\hat{\lambda}^*)^2 \quad (17)$$

where the physical scalar coupling constant $\lambda^* \equiv G_\mu^*(m_H^*)^2/\sqrt{2}$. To get the \overline{MS} -version of (17) we have calculated a relation $\lambda^*[\lambda^{MS}]$. Following the same strategy as used above for $\alpha_t^*[\alpha_t^{MS}]$ we obtain on the one-loop level (see details in [12]):

$$\hat{\lambda}^* = \hat{\lambda}^{MS} \left(1 - \left[25 - 3\pi\sqrt{3} + 12\ln(\mu^2/m_H^2) \right] \hat{\lambda}^{MS} \right) \quad (18)$$

Hence the \overline{MS} -expression for $\Delta\Gamma$ numerically reads as

$$\Delta\Gamma = 1 + 2.1172\hat{\lambda}^{MS} - 37.8167(\hat{\lambda}^{MS})^2 \quad (19)$$

for $\mu^2 = m_H^2$. The coefficient of $\hat{\lambda}^2$ appears to be larger in \overline{MS} than in MOS -scheme.

In conclusion, we have found that while in the on-shell subtraction scheme the QCD $\mathcal{O}(\alpha_t\alpha_s)$ -correction dominates and there is sensitivity to the heavy top quark mass, in the

\overline{MS} -scheme the EW $\mathcal{O}(\alpha_t^2)$ -correction dominates and there is sensitivity to the heavy Higgs mass.

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Figure Captions

Fig. 1. Tad-pole diagrams responsible for the shift of the vacuum expectation value v in \overline{MS} -scheme from its tree-level value: dashed line - Higgs boson loop, solid line - t -quark loop.

Fig. 2. The ratio m_t^*/m_t^{MS} (solid line) and the ratio α_t^*/α_t^{MS} (dashed line) as functions of r in the interval of r corresponding to the Higgs masses $m_H \simeq \{60GeV, 1TeV\}$.

Fig. 3. QCD (dashed line) and EW (solid line) corrections to the one-loop expression for $\Delta\rho^{(1)}$ as a percent of $\Delta\rho^{(1)}$ in the \overline{MS} -scheme.

Fig. 4. $\Delta\rho$ as a function of r on the 2-loop level in the MOS -scheme (dashed line) and \overline{MS} -scheme (solid line). Dotted line corresponds to using \overline{MS} -scheme in QCD calculations only (eq. (7)).

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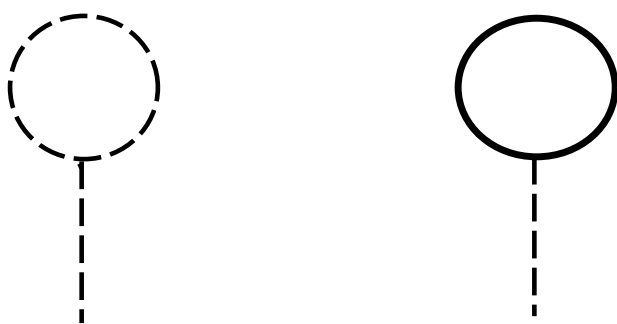
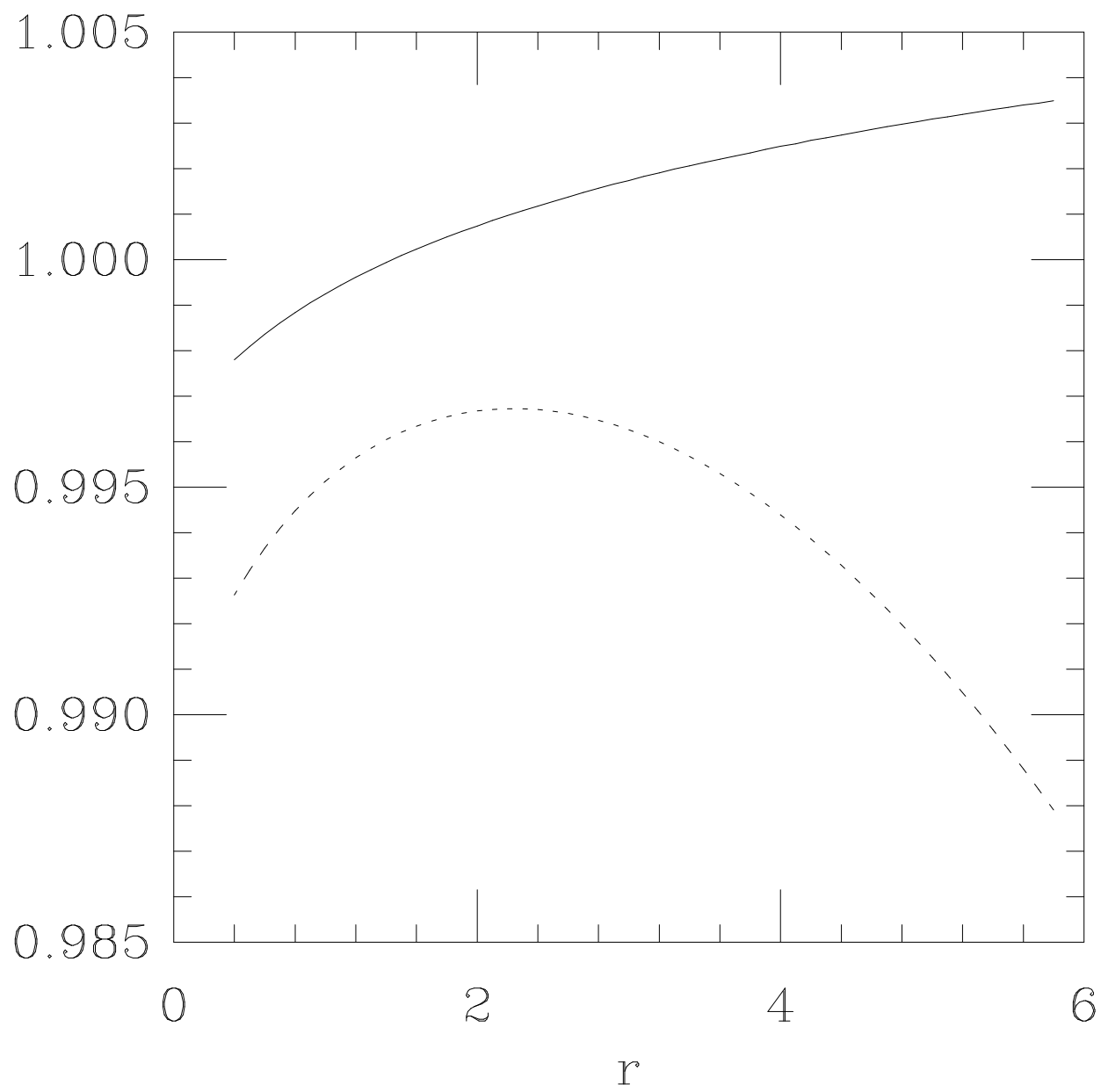


Fig. 1

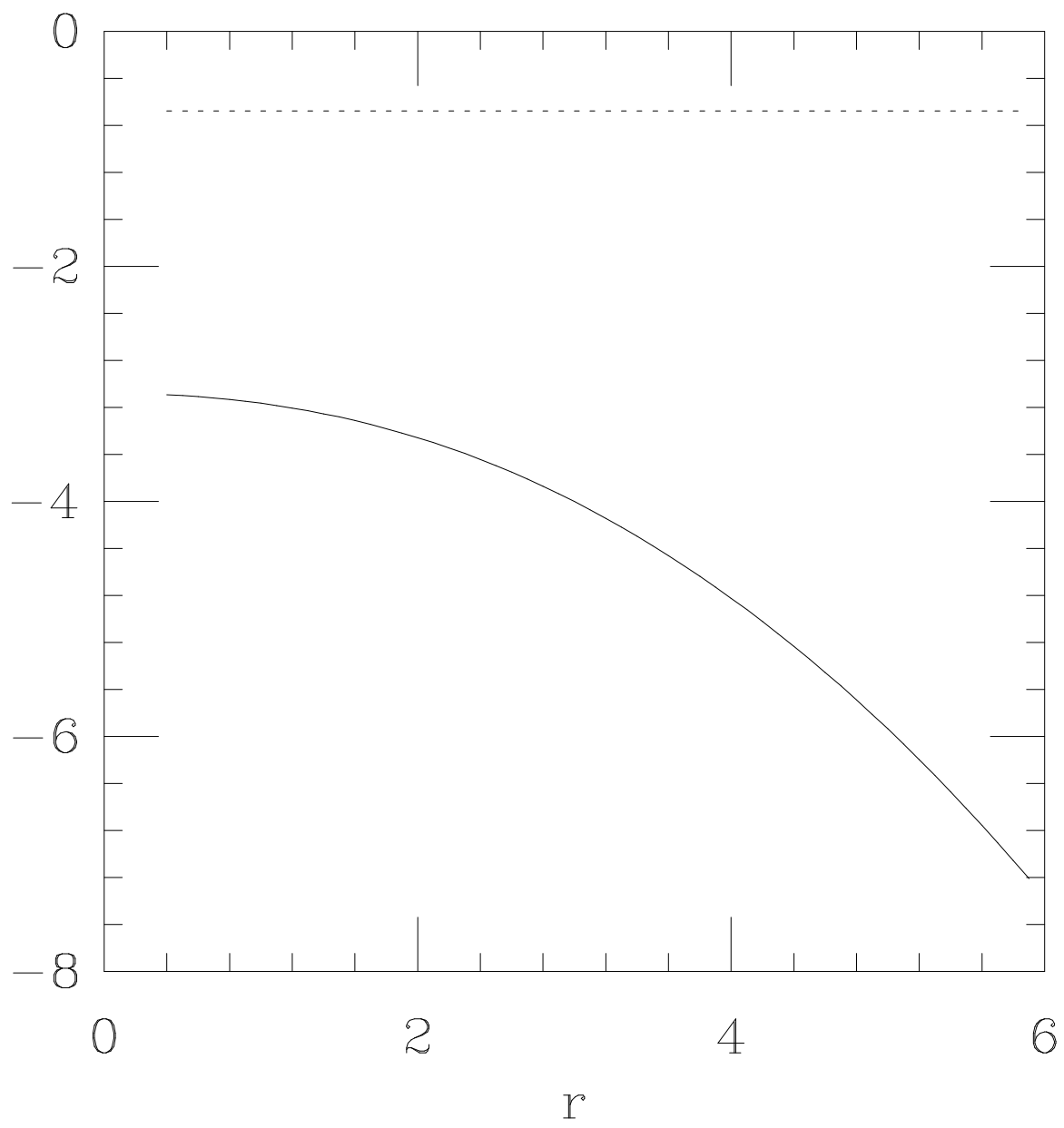
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